Single Pure – Quadratics In Disguise

To spot a quadratic in disguise you are looking for an equation where the power on one of the variables is twice that on the other. For example

$$(\text{whatever})^{18} + 4(\text{whatever})^{9} - 5.$$

This can then factorise to

$$((\text{whatever})^{9} + 5)((\text{whatever})^{9} - 1),$$

or you can complete the square to

$$((\text{whatever})^{9} + 2)^{2} - 9.$$

Solve the following:

1. $x^4 - 13x^2 + 36 = 0.$ $\boxed{x = \pm 2 \text{ or } x = \pm 3}$
2. $x^4 - 15x^2 - 16 = 0.$ $\boxed{x = \pm 4}$
3. $x^4 + 5x^2 + 6 = 0.$ $\boxed{\text{No real solutions}}$
4. $x^6 + 7x^3 = 8.$ $\boxed{x = -2 \text{ or } x = 1}$
5. $2x^4 = x^2 + 1.$ $\boxed{x = \pm 1}$
6. $x + 3 = 4\sqrt{x}.$ $\boxed{x = 1 \text{ or } x = 9}$
7. $12x^4 = 2x^2 + 4.$ $\boxed{x = \pm \frac{\sqrt{6}}{3}}$
8. $2(\sin x)^2 + \sin x - 1 = 0$ in range $0 < x < 360$. $\boxed{x = 270 \text{ or } x = 30 \text{ or } x = 150}$
9. $\frac{4x^4+144}{73} = x^2.$ $\boxed{x = \pm \frac{3}{2} \text{ or } x = \pm 4}$
10. $2(\cos x)^2 + (\cos x) = 6.$ $\boxed{\text{No real solutions}}$
11. $x = 2\sqrt{x} + 3.$ $\boxed{x = 9}$
12. $6x^{2/3} + 5x^{1/3} - 4 = 0.$ $\boxed{x = \frac{1}{8} \text{ or } x = -\frac{64}{27}}$
13. $(x^2 - 4x + 1)^2 + (x^2 - 4x + 1) - 12 = 0.$ $\boxed{x = 2 \pm \sqrt{6}}$
14. $2\theta + 15 = 11\sqrt{\theta}.$ $\boxed{\theta = 9 \text{ or } \theta = \frac{25}{4}}$
15. $x^2 + \frac{72}{x^2} = 17.$ $\boxed{x = \pm 3 \text{ or } x = \pm 2\sqrt{2}}$
16. $\sqrt{z} - 2\sqrt[4]{z} = 3.$ $\boxed{z = 81 \text{ (only)}}$
17. $2^{2x} - 12 \times 2^x + 32 = 0.$ $\boxed{x = 2 \text{ or } x = 3}$
18. $t = 4\sqrt{t} + 1.$ $\boxed{t = 9 \pm 4\sqrt{5}}$
19. $2^{2x} + 8 = 9 \times 2^x.$ $\boxed{x = 0 \text{ or } x = 3}$
20. $2\sqrt{x} + \frac{9}{\sqrt{x}} = 9.$ $\boxed{x = \frac{9}{4} \text{ or } x = 9}$
21. $\left(\frac{1}{x}\right)^2 + 1 = 8\left(\frac{1}{x}\right).$ [Do this question in two ways.] $\boxed{x = 4 \pm \sqrt{15}}$
22. $2(\cos\theta)^2 = 8\cos\theta + 21.$ $\boxed{\text{No real solutions}}$
23. $x^8 + 4x^4 = 3.$ $\boxed{x = \pm \sqrt[4]{-2 + \sqrt{7}}}$
24. $2^{2x} + 1 = 2^{x+1}.$ $\boxed{x = 0}$
25. $2^{2x} + 128 = 3 \times 2^{x+3}.$ $\boxed{x = 3 \text{ or } x = 4}$
26. $81 + 3^{2x+1} = 4 \times 3^{x+2}.$ $\boxed{x = 2 \text{ or } x =}$
27. $a^{2x} + a^4 = a^{x+1} + a^{x+3}.$ $\boxed{x = 1 \text{ or } x = 3}$

Only attempt the following if you have studied logarithms.

28. $2^{2x} - 13 \times 2^x + 42 = 0.$ $\boxed{x = \log_2 7 \text{ or } x = \log_2 6}$
29. $4^{2x} - 9 \times 4^x + 14 = 0.$ $\boxed{x = \log_4 7 \text{ or } x = \frac{1}{2}}$
30. $3^{2x} + 10 = 7 \times 3^x.$ $\boxed{x = \log_3 2 \text{ or } x = \log_3 5}$
31. $2^{2x} - 5 \times 2^{x+1} + 25 = 0.$ $\boxed{x = \log_2 5 \text{ (repeated)}}$
32. $3^{2x} - 3^{x+2} + 20 = 0.$ $\boxed{x = \log_3 4 \text{ or } x = \log_3 5}$